A long-standing important problem in number theory is to prove that there are infinitely many twin primes $p$ and $p + 2$. This would show that there are very small gaps between primes; indeed, 2 is the smallest possible gap between primes (apart from 2 and 3). While this problem has attracted considerable attention, it still remains seemingly intractable. We may weaken the question, and ask whether there are gaps between primes substantially smaller than the ”usual” gap. The prime number theorem tells us that there are about $x/\log x$ primes below $x$. So if we take primes around size $x$, the gap between consecutive primes is usually around size $\log x$. Recently, Goldston, Pintz and Yildirim have made a spectacular advance in prime number theory and showed that there are primes around size $x$ with the gap to the next prime being smaller than $\epsilon \log x$ for any given positive number $\epsilon$. Moreover, they showed that if certain standard conjectures on primes in arithmetic progressions are true, then there would be bounded gaps between consecutive primes. I will explain their results and some of the ideas that go into the proof. (Received September 28, 2005)