

1014-11-217

**Mary E. Flahive\*** (flahive@math.oregonstate.edu) and **Richard T. Bumby** (bumby@math.rutgers.edu). *Inhomogeneous Diophantine approximation for irrationals with quasi-periodic continued fractions*. Preliminary report.

For  $\theta$  and  $\phi$  with  $q\theta - \phi \notin Z$  for integral  $q$ , the inhomogeneous approximation constant is

$$M(\theta, \phi) = \inf_{|q| \rightarrow \infty} \{|q| |q\theta - \phi|\}.$$

Minkowski proved  $M(\theta, \phi) \leq 1/4$ , and J. H. Grace [Proc London Math Soc 17 (1918), 316–319] constructed  $\theta$  with  $M(\theta, 1/2) = 1/4$ . We consider the case when  $\phi \in Q(\theta)$  and the sequence of partial quotients of  $\theta$  eventually is  $\phi_1(0), \dots, \phi_J(0), \dots, \phi_1(i), \dots, \phi_J(i), \dots$ , where  $\{\phi_j(i)\}$  are arithmetic progressions. We extend work of Takao Komatsu which used many different types of continued fractions to calculate  $M(e^{2/s}, \phi)$  for  $\phi \in Q(\theta)$ . Here we use regular simple continued fractions and a modification of Grace's method to generalize and obtain new results for the case when  $\theta = e^{2/s}$ . Among these are a characterization of pairs  $\theta, \phi$  (restricted as above) for which  $M(\theta, \phi) = 0$  and a characterization of all  $\phi = \frac{r\theta+m}{n}$  with  $M(\theta, \phi) < 1/n^2$ . The work uses a compactness theorem to relate  $M(\theta, \phi)$  to the smallest value of the product of two linear expressions with rational coefficients. (Received August 25, 2005)