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Discrepancy of Fractions with Divisibility Constraints. Preliminary report.

Niederreiter showed the discrepancy D_N of the Farey series \mathcal{F}_Q of order Q satisfies $D_N \asymp 1/\sqrt{N_Q}$, where $N_Q = \#\mathcal{F}_Q$. Recently Dress proved the striking fact that $D_N = 1/Q$. In relation to their works, we provide bounds for the discrepancy of \mathcal{F}_Q with \mathcal{B} -free denominators satisfying divisibility constraints; i.e., given a sequence \mathcal{B} of positive integers $1 < b_1 < b_2 < \dots$ such that

$$\sum_{k=1}^{\infty} \frac{1}{b_k} < \infty \text{ with } \gcd(b_k, b_j) = 1 \text{ for } k \neq j,$$

a number n is called \mathcal{B} -free if no element b_k of \mathcal{B} divides n . Thus for any $Q \geq 1$ and any such set \mathcal{B} , consider for any $q \equiv u \pmod{k}$, with $k, u \geq 1$,

$$\mathcal{F}_{Q,u,k,\mathcal{B}} = \left\{ \frac{a}{q} \in \mathcal{F}_Q; q \equiv u \pmod{k}, q \text{ is } \mathcal{B}\text{-free} \right\}$$

and define

$$D_{N_{Q,u,k,\mathcal{B}}} = \sup_{0 \leq \alpha \leq 1} \left| \frac{A_{Q,u,k,\mathcal{B}}(\alpha)}{N_{Q,u,k,\mathcal{B}}} - \alpha \right|,$$

where $A_{Q,u,k,\mathcal{B}}(\alpha) = \#(\mathcal{F}_{Q,u,k,\mathcal{B}} \cap [0, \alpha])$ and $N_{Q,u,k,\mathcal{B}} = \#\mathcal{F}_{Q,u,k,\mathcal{B}}$. We will show that

$$D_{N_{Q,u,k,\mathcal{B}}} \underset{k,\mathcal{B}}{\asymp} \frac{1}{Q}.$$

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