Factoring Generalized Repunits

A repunit $R_n$ is an integer written in decimal form as a string of $n$ ones. More than twenty years ago, W. M. Snyder extended the notion of a repunit to one in which for some integer $b > 1$, $R_n(b)$ has a $b$-adic expansion consisting of only ones; that is, $R_n(b) = \sum_{i=1}^{n-1} b^i = \frac{b^n - 1}{b-1}$, where $n > 0$. Examples include the Mersenne numbers, $M_n = 2^n - 1 = 1 + 2^1 + 2^2 + \ldots + 2^{n-1}$, for $n \geq 2$. Snyder’s admitted objective was to apply algebraic number theory in cyclotomic fields in order to determine the pairs of integers $(n, b)$ under which $R_n(b)$ has a prime divisor congruent to 1 modulo $n$. Specifically, he proved that $R_n(b)$ has a prime divisor congruent to 1 (mod $n$) if and only if either $n \neq 2$, or $n = 2$ and $b \neq 2^e - 1$, for all $e > 1$. In this talk, we shall demonstrate how this result follows from theory pertaining to the Lucas sequences. (Received September 22, 2005)