Hilbert’s Tenth Problem in its original form was to find an algorithm to decide, given a polynomial equation $f(x_1, \ldots, x_n) = 0$ with coefficients in the ring $\mathbb{Z}$ of integers, whether it has a solution with $x_1, \ldots, x_n \in \mathbb{Z}$. Matiyasevich proved that no such algorithm exists, i.e. Hilbert’s Tenth Problem is undecidable. Since then, analogues of this problem have been studied by asking the same question for polynomial equations with coefficients and solutions in other commutative rings.

Let $k$ be a subfield of a $p$-adic field. We will prove that Hilbert’s Tenth Problem for function fields of varieties over $k$ is undecidable. (Received September 26, 2005)