A curve over a field $k$ is *pointless* if it has no $k$-rational points. We show that there exist pointless genus-3 hyperelliptic curves over a finite field $\mathbb{F}_q$ if and only if $q \leq 25$, that there exist pointless smooth plane quartics over $\mathbb{F}_q$ if and only if either $q \leq 23$ or $q = 29$ or $q = 32$, and that there exist pointless genus-4 curves over $\mathbb{F}_q$ if and only if $q \leq 49$.

We use a variety of techniques to show that pointless curves of a given genus over a given field do not exist. To show that pointless curves *do* exist over a given field, we provide explicit examples. I will explain how we use Galois theory to help us find these explicit examples quickly. (Received September 26, 2005)