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A curve over a field  $k$  is *pointless* if it has no  $k$ -rational points. We show that there exist pointless genus-3 hyperelliptic curves over a finite field  $\mathbb{F}_q$  if and only if  $q \leq 25$ , that there exist pointless smooth plane quartics over  $\mathbb{F}_q$  if and only if either  $q \leq 23$  or  $q = 29$  or  $q = 32$ , and that there exist pointless genus-4 curves over  $\mathbb{F}_q$  if and only if  $q \leq 49$ .

We use a variety of techniques to show that pointless curves of a given genus over a given field do not exist. To show that pointless curves *do* exist over a given field, we provide explicit examples. I will explain how we use Galois theory to help us find these explicit examples quickly. (Received September 26, 2005)