Let $S = k[x_1, \ldots, x_n]$ be a polynomial ring over a field $k$. We study the graded Betti numbers of homogeneous ideals $I$ which contain the squares $P = (x_1^2, \ldots, x_n^2)$. Our main result is the lex-plus-powers conjecture for such ideals (due to Herzog and Hibi, and later in a more general form to Evans): We prove that if $k$ has characteristic zero, and $L \subset S$ is a squarefree lexicographic ideal such that $I$ and the lex-plus-squares ideal $L + P$ have the same Hilbert function, then the graded Betti numbers of $L + P$ are greater than or equal to those of $I$. (Received September 28, 2005)