Let $D$ be an integral domain with unit group $U(D)$ and $D^\# = D \setminus (U(D) \cup \{0\})$; let $\tau$ be a (symmetric) relation on $D^\#$. For $a \in D^\#$, we define a factorization $a = \lambda a_1 \cdots a_n$ to be a $\tau$-factorization of $a$ if $\lambda \in U(D)$, $a_i \in D^\#$, and $a_i \tau a_j$ for $i \neq j$. We examine the $\tau$-factorization properties which arise from defining $a \tau b$ iff $a \equiv b$ modulo $n$ for $a, b \in \mathbb{Z}$. (Received September 21, 2005)