Let $J = \{j_1, \ldots, j_t\}$ be a subset of $[n] = \{1, \ldots, n\}$, and let $m_J$ be the ideal $(x_{j_1}, \ldots, x_{j_t})$ in $R = k[x_1, \ldots, x_n]$. We will discuss intersections of Veronese ideals, which are ideals of the form $I = m_{J_1}^{a_1} \cap \cdots \cap m_{J_s}^{a_s}$, where the $J_i$ are subsets of $[n]$. These ideals arise naturally in a number of settings, including as ideals of some sets of fat points, as ideals of tetrahedral curves, and in combinatorics when all $a_i = 1$. We will identify a number of cases in which these ideals are componentwise linear by using the theory of polymatroidal ideals, and we will discuss some applications in algebra and combinatorics. (Received September 12, 2005)