Let $D$ and $T$ be domains, let $I$ be an ideal of $T$, let $\varphi : T \to T/I$ be the canonical map, and let $R = \varphi^{-1}(D)$. We attempt to characterize when $R$ has certain arithmetic properties, obtaining reasonably satisfactory results in several cases. For example, extending a result of Mimouni, we show that $R$ is a Prüfer domain if and only if $D$ and $T$ are Prüfer domains, $I$ is a prime ideal of $T$, and $D$ and $T/I$ have the same quotient fields. Some of our results generalize facts known to hold for the $A + XB[X]$- and $D + XD_{S}[X]$-constructions. (Received September 26, 2005)