A matrix $A \in M_n(C)$ is called *Hermitian* if $A = A^*$. A Hermitian matrix with nonnegative eigenvalues are called *positive semi-definite (PSD)* matrices. Given a Hermitian matrix $A$ we associate a simple, undirected graph $G$ with $V(G) = \{1 \cdots n\}$ and edges $E(G) = \{(i, j) \mid a_{ij} \neq 0, i \neq j\}$. The graph is independent of the diagonal entries of $A$. The *minimum positive semi-definite (PSD) rank* of $G$, denoted $msr(G)$, is the minimum rank of $A$ where $A$ varies over all PSD matrices with graph $G$.

For a simple connected graph $G$ we define the *tree size of $G$*, denoted $ts(G)$, as the number of vertices in the maximum induced tree in $G$, and the *clique cover number*, denoted $c(G)$, as the smallest number of cliques needed to cover all the edges in $G$.

In this paper we present some results on the minimum PSD rank of some classes of graphs, including bipartite graphs, non-chordal graphs for which $msr(G) = c(G)$, and graphs for which $msr(G) = ts(G) - 1$. Also, we present some additive properties of $msr(G)$ for a graph $G$ that can be identified as overlapping sum of two subgraphs by considering the effect of edge cancellation on the graph.

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