Let $V$ be a finite dim. vector space over an alg. closed field $K$. Let $q$ be a nonzero scalar in $K$ that is not a root of unity.

Consider a pair on linear maps $A : V \to V$, $A^* : V \to V$ which satisfy the following:

1. There exists a basis for $V$ with respect to which the matrix representing $A$ is diagonal and the matrix representing $A^*$ is irreducible tridiagonal.

2. There exists a basis for $V$ with respect to which the matrix representing $A^*$ is diagonal and the matrix representing $A$ is irreducible tridiagonal.

We call such a pair a Leonard pair on $V$. We assume there exist nonzero scalars $a, b, c$ in $K$ such that the eigenvalues of $A$ (resp. $A^*$) are $aq^{2i-d}$ (resp. $bq^{2i-d} + cq^{d-2i}$) for $0 \leq i \leq d$. We discuss how such Leonard pairs are divided into two families. For one family we use the Leonard pair to construct two irreducible $U_q(\widehat{sl}_2)$-module structures on $V$ and describe how these modules are related to the actions of $A$ and $A^*$. For the other family we use the Leonard pair to construct an irreducible $U_q(\mathfrak{sl}_2)$-module structure on $V$ and describe how this module is related to the actions of $A$ and $A^*$. (Received September 26, 2005)