Peter Heinzner and Gerald Schwarz* (schwarz@brandeis.edu), Department of Mathematics, Mail Stop 050, PO Box 549110, Waltham, MA 02454-9110. Cartan decomposition of the moment map.

Let $Z$ be a complex space with a holomorphic action of the complex group $U^C$, where $U^C$ is the complexification of the compact Lie group $U$. We assume that $Z$ admits a smooth $U$-invariant Kähler structure and a $U$-equivariant moment mapping $\mu: Z \rightarrow u^*$. We assume that $G \subset U^C$ is a closed subgroup such that the Cartan decomposition $U^C \simeq U \times iu$ induces a decomposition $G \simeq K \times p$ where $K = U \cap G$ and $p \subset iu$ is an $(\text{Ad} \, K)$-stable linear subspace. By restriction we have an induced “moment” mapping $\mu_p: Z \rightarrow (ip)^*$. We define $\mathcal{M}$ to be the set of zeroes of $\mu_p$ and we have the set of semistable points $\mathcal{S}_G(\mathcal{M}) := \{z \in Z; G \cdot z \cap \mathcal{M} \neq \emptyset\}$. Then $\mathcal{M}$ is the analogue of the Kempf Ness set for linear actions. We establish the existence of a quotient $\mathcal{S}_G(\mathcal{M})/G$ and we show that $\mathcal{M}/K \simeq \mathcal{S}_G(\mathcal{M})/G$. We also establish a version of Luna’s slice theorem as well as a version of the Hilbert-Mumford criterion. A global slice theorem is proved for proper $G$-actions.