M. Antonowicz and A.P. Fordy introduced in 1988 the second order polynomial eigenvalue problem which contains the arbitrary power $n$ of the eigenvalue lambda. The Harry Dym equation corresponds to $n = 1$, and the Coupled Harry Dym equation corresponds to $n = 2$. In both cases, we have obtained the associated finite dimensional Integrable Hamiltonian systems (FDIHS) using the nonlinearization method and Bargmann constraints a long time ago. For general, arbitrary, positive integer $n$, it is much more difficult to find the associated finite dimensional integrable Hamiltonian system. In this paper, we present the way to find the FDIHS systems associated the above polynomial eigenvalue problem with arbitrary $n$. (Received September 26, 2005)