Goong Chen* (gchen@math.tamu.edu), Department of Mathematics, Texas A&M University, College Station, TX 77843. Mathematical Analysis of the Fourier Spectrum of Chaos with Applications to the Chaotic Vibration of the Wave Equation with a Nonlinear Boundary Condition.

Consider an interval map $f : I \to I$. If $f$ is a chaotic map, then what kind of special properties do the Fourier coefficients of the iterates of $f$ have? Many physicists and engineers seem to have run numerical simulations and made conclusions about the occurrence of chaos from the Fourier (or, what they call, power) spectrum. But interestingly, no quantitative criteria or assessments were obtained in definitive terms in the literature, to the best of our knowledge.

In this talk, we present necessary and sufficient conditions between Fourier coefficients of iterates and the presence of chaos. These conditions involve certain special properties of $c(n, k)$, where $c(n, k)$ is the $k$-th Fourier coefficient of $f^n$ (the $n$-th iterate of the interval map $f$). We utilize the exponential growth property of the total variations of $f^n$ and the $H^1$ Sobolev space norm of $f^n$ to obtain the desirable estimates.

From the above properties of interval maps, we readily apply them to the chaotic vibration of the wave equation with a nonlinear boundary condition of the van der Pol type. Concrete examples will be illustrated to show the tightness of our theorems. (Received September 21, 2005)