

1014-37-221

**Tiancheng Ouyang** and **Zhifu Xie\*** (zhifu@math.byu.edu), 292TMCB, Department of Mathematics, Brigham Young University, Provo, UT 84602. *Index Theory for Symplectic Paths and the Stability of Periodic Solutions for N-body Problem.*

We apply index theory for symplectic paths introduced by Y. Long to study the stability of a periodic solution  $x$  for a Hamiltonian system, particular for N-body problem. We first study the topological properties of the symplectic group  $Sp(2n) = \{M \in GL(\mathbb{R}^{2n}) | M^T J M = J\}$ , where  $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$ ,  $I$  is the identity matrix. An index function is defined for each symplectic path  $\gamma(t)$  starting from identity. Because the fundamental solution of the linearized Hamiltonian system along the periodic solution  $x$  is a symplectic path (called the associated symplectic path of the periodic solution), an index  $ind(x)$  is defined for  $x$  by the index of its associated symplectic path. Then the periodic solution  $x$  for a two dimensional Hamiltonian system is linear stable if and only if its index  $ind(x)$  is an odd integer. The index can also be expressed in terms of Morse index. For higher dimension (4-dimension), some necessary conditions for stability and instability are established. This is a new way to study stability. These theorems are applied to study the stability problem of Isosceles three body problem proposed by Daniel Offin and figure-eight solution found by A. Chenciner and R. Montgomery. (Received August 26, 2005)