One of the most useful descriptions of a physical system is provided by the statistics of time series of observations on
the system. We describe recent work on establishing the almost sure invariance principle (ASIP) for Hölder observations
on a broad class of non-uniformly hyperbolic dynamical systems. The ASIP essentially states that there is a Brownian
motion that approximates measurements along almost every trajectory of the system. Suppose \( f : X \to X \) is a map and
\( \phi : X \to \mathbb{R} \) is an observable satisfying the ASIP. The ASIP implies that aspects of the asymptotic behavior of Brownian
motion translate to the sequence of random variables \( \{ \sum_{j=0}^{N-1} \phi \circ f^j \} \). The ASIP implies weaker distributional results such
as the central limit theorem as well as bounds on the rate of growth such as the law of the iterated logarithm. (Received
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