Absolute equal distribution of the eigenvalues of discrete Sturm-Liouville problems.

We consider the asymptotic relationship as \( n \to \infty \) between the eigenvalues \( \lambda_1 \leq \cdots \leq \lambda_n \) and \( \mu_1 \leq \cdots \leq \mu_n \) of the Sturm–Liouville problems defined for \( n \geq 2k + 1 \) by

\[
\sum_{\ell=0}^{k} (-1)^{\ell} \Delta^\ell (r_{in}(i-\ell)\Delta^\ell x_{i-\ell}) = \lambda\phi_{in}x_i, \quad 1 \leq i \leq n,
\]

and

\[
\sum_{\ell=0}^{k} (-1)^{\ell} \Delta^\ell (s_{in}(i-\ell)\Delta^\ell x_{i-\ell}) = \mu\psi_{in}x_i, \quad 1 \leq i \leq n,
\]

where \( x_i = 0 \) if \(-k + 1 \leq i \leq 0\) or \( n + 1 \leq i \leq n + k\), all quantities are real, and \( \phi_{in}, \psi_{in} > 0, \quad 1 \leq i \leq n, \quad n \geq 2k + 1 \). We give conditions implying that

\[
\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} |F(\lambda_{in}) - F(\mu_{in})| = 0
\]

for all \( F \in C(-\infty, \infty) \) such that \( \lim_{x \to -\infty} F(x) \) and \( \lim_{x \to \infty} F(x) \) exist (finite). (Received August 05, 2005)