Sampling and Interpolation of Two-Band Signals.

We address the problem of construction of sampling and interpolation sequences for multi-band signals, that is signals whose spectra lie in a finite union of disjoint intervals: $E = \bigcup_{j=1}^{N} I_j$, $I_j = [a_j, b_j]$, $0 = a_1 < b_1 < a_2 < b_2 < \ldots < a_N < b_N$. Recently several papers have related the production of sampling and interpolating sequences for multi-band signals to the solution of certain kinds of Wiener-Hopf equations. Our approach is based on connections between exponential Riesz bases and controllability of distributed parameter systems: the problem of sampling is translated into one of finding an exponential Riesz basis in $L^2(E)$, and a Riesz basis is obtained by finding eigenfrequencies of an auxiliary dynamical system that must be exactly controllable. The problem of controllability of that system can be reduced to the invertibility of a convolution operator which is isomorphic to a certain Wiener-Hopf operator. For the case of two-band signals we derive an operator whose invertibility is equivalent to the existence of a sampling and interpolating sequence, and prove invertibility of this operator. The approach allows also to construct a large set of sampling and interpolation sequences. (Received September 27, 2005)