In this talk we will survey recent progress on the Lipschitz extension problem, which asks for conditions on a pair of metric spaces $X,Y$ such that every $Y$-valued Lipschitz map on a subset of $X$ can be extended to all of $X$ with only a bounded multiplicative loss in the Lipschitz constant. We will focus on various probabilistic methods which have been used in the proofs of such results, including random projections, random partitions of metric spaces, and the analysis of Markov chains in metric spaces. (Received September 15, 2005)