We define a new topology on $C(X)$ and investigate its properties. We call this topology the *regular topology* (or *r*-topology for short) on $C(X)$ and denote it by $C_r(X)$. This topology is finer than the $m$-topology, which in turn is finer than the uniform topology. We investigate and determine for which Tychonoff spaces $X$, $C_r(X)$ has specific topological properties. In particular, we shall discuss when this topology is first countable and make an attempt at characterizing the character of $C_r(X)$. The answers are somewhat more complex than for the $m$-topology on $C(X)$.

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