Both the mod $p$ cohomology of spaces and mod $p$ algebras of invariants are unstable algebras over the Steenrod algebra $A$. Some arise in both settings, like the cohomology of classifying spaces $BO, BU$, while others, such as the Dickson algebras of $GL_n$ invariants, emerge primarily as invariants. While such $A$-algebras often seem intractable as $A$-modules, we find that they often have simple presentations as unstable $A$-algebras via minimal generators and relations. We work with unstable $A$-modules using the Kudo-Araki-May (K-A-M) algebra $K$ of certain ‘lower’ operations as originally defined by Steenrod, before their conversion to ‘Steenrod operations’. $K$ is particularly suited to describing unstable modules.

We also find that, for $p$ odd, Steenrod’s original operations encompassed more than those converted for topology. This larger algebraic K-A-M algebra may yield expanded connections to invariants. In particular, Madsen and May proved that together the Dickson algebras are the dual of the topological K-A-M algebra $K$; we will discuss how invariants under subgroups of $GL_n$ may yield duals coming from the larger algebraic K-A-M algebra. (Received September 26, 2005)