Towards the continuous Hamiltonian dynamical systems.

We first introduce the notion of the \((C^0)\) Hamiltonian topology on the space of Hamiltonian paths, and on the group of Hamiltonian diffeomorphisms respectively. We then define a group denoted by \(\text{Hameo}(M, \omega)\) consisting of Hamiltonian homeomorphisms that we also define. We prove that \(\text{Ham}^{(1,1)}(M, \omega) \subset \text{Hameo}(M, \omega) \subset \text{Sympeo}(M, \omega)\) where \(\text{Sympeo}(M, \omega)\) is the group of symplectic homeomorphisms and \(\text{Ham}^{(1,1)}(M, \omega)\) is the set of homeomorphisms obtained by the time-one maps of \(C^{(1,1)}\) time-dependent Hamiltonian functions. We prove that \(\text{Hameo}(M, \omega)\) is a normal subgroup of \(\text{Sympeo}(M, \omega)\) which is path-connected and so contained in the identity component \(\text{Sympeo}_0(M, \omega)\) of \(\text{Sympeo}(M, \omega)\).

In the case of two dimensional compact surfaces, we prove that the mass flow of any element from \(\text{Hameo}(M, \omega)\) vanishes, which in turn implies that \(\text{Hameo}(M, \omega)\) is strictly smaller than the identity component of the group of area preserving homeomorphisms when \(M \neq S^2\). For the case of \(S^2\), we conjecture that the same is still true. (The latter group turns out to coincide with \(\text{Sympeo}_0(M, \omega)\) for two dimensional surface \(M\).) (Received September 16, 2005)