Up until the mid 70s the kind of spectra most people had in mind in the context of theory of Schrodinger operators were spectra occurring for periodic potentials and for atomic and molecular Hamiltonians. Then evidence started to build up that “exotic” spectral phenomena such as singular continuous, Cantor, and dense point spectrum do occur in mathematical models that are of substantial interest to theoretical physics. One area where such exotic phenomena are particularly abundant is quasiperiodic operators. They feature a competition between randomness (ergodicity) and order (periodicity), which is often resolved on a deep arithmetic level. Mathematically, the methods involved include a mixture of ergodic theory, dynamical systems, probability, functional and harmonic analysis. The interest in those models was enhanced by strong connections with some major discoveries in physics, such as integer quantum Hall effect, experimental quasicrystals, and quantum chaos theory. Quasiperiodic operators provide central or important models for all three. We will review the ideas that were important for recent solutions of several open problems in this subject.

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