Finite Dimensional Approximations to Wiener Measure on a Compact Manifold with Positive Curvature.

Let $H(M)$ be the Hilbert manifold of finite energy paths into a compact Riemannian manifold, $M$. We will equip $H(M)$ with its natural $G^1$ metric. Given a partition, $\mathcal{P}$ of $[0, 1]$, let $H_\mathcal{P}(M)$ be the finite dimensional Riemannian submanifold of $H(M)$ consisting of piecewise geodesic paths adapted to $\mathcal{P}$. Under certain curvature restrictions on $M$, it is shown that

$$\frac{1}{Z_\mathcal{P}}e^{-\frac{1}{2}E(\sigma)}dVol_{H_\mathcal{P}}(\sigma) \to \rho(\sigma)d\nu(\sigma) \text{ as } \text{mesh}(\mathcal{P}) \to 0,$$

where $Z_\mathcal{P}$ is a “normalization” constant, $E : H(M) \to [0, \infty)$ is the energy functional, $Vol_{H_\mathcal{P}}$ is the Riemannian volume measure on $H_\mathcal{P}(M)$, $\nu$ is Wiener measure on continuous paths on $M$, and $\rho$ is a certain density determined by the curvature tensor of $M$. (Received September 26, 2005)