The Cayley graph $Cay(Z_a \times Z_b : \{(1,0), (0,1)\})$ is the digraph with vertices the elements of $Z_a \times Z_b$ and with a directed edge from vertices $\alpha$ to $\beta$ if $\alpha = \beta + x$ for $x \in \{(1,0), (0,1)\}$. Given a subgroup $G$ of $Z_a \times Z_b$, we provide necessary and sufficient conditions for $Cay(Z_a \times Z_b - G : \{(1,0), (0,1)\})$ to have a Hamiltonian cycle, where the vertices of $G$ and their associated edges are deleted from $Cay(Z_a \times Z_b : \{(1,0), (0,1)\})$. In addition, for a subgroup $Z_c$ of $Z_a$ and a subgroup $Z_d$ of $Z_b$, we give necessary and sufficient conditions for $Cay(Z_a \times Z_b : \{(1,0), (0,1)\})$ to have a spanning closed walk passing through every vertex in $Z_c \times Z_d$ exactly twice and all other vertices exactly once. We also determine precisely when $Cay(Z_a \times Z_b - R_{k_1 \times k_2} : \{(1,0), (0,1)\})$ has a Hamiltonian cycle, where $R_{k_1 \times k_2}$ is the set of vertices in an $k_1 \times k_2$ rectangle. (Received September 21, 2006)