Let $G$ be the Klein 4-group. For a Galois extension $E/F$ of fields with $\text{Gal}(E/F) \simeq G$ we consider the $F_2\text{Gal}(E/F)$-module $J(E/F)$ of square classes of units of $E$. We determine the structure of $J(E/F)$ in terms of arithmetic invariants of $E/F$, and we show that only finitely many indecomposable $F_2G$-modules may appear as summands. We also determine the structure of $E/\wp(E)$ in characteristic 2 and show that only two indecomposable $F_2G$-modules appear as summands. (Received July 24, 2006)