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Joshua Buckner* (joshua_buckner@baylor.edu), Baylor University, Mathematics Department,
One Bear Place #97328, Waco, TX 76798-7328. *Zassenhaus Rings of Finite Rank.*

Let S be an integral domain, R_S an S algebra, and \mathcal{F} a family of left ideals of R . We define $End_S(R, \mathcal{F}) = \{\varphi \in End_S(R^+) : \forall X \in \mathcal{F}(\varphi(X) \subset X)\}$. In 1967, H. Zassenhaus proved that if R is a ring such that R^+ is free of finite rank, then there is a left R module M such that $R \subset M \subset \mathbb{Q}R$ and $End_{\mathbb{Z}}(M) = R$. This motivates the following definitions: We call $R_{\mathbb{Z}}$ a Zassenhaus ring with module M if the conclusion of Zassenhaus' result holds for the ring R and module M . It is easy to see that if $R_{\mathbb{Z}}$ is a Zassenhaus ring then R has a family \mathcal{F} of left ideals such that $End_{\mathbb{Z}}(R, \mathcal{F}) = R$. (If \mathcal{F} has this property, then we call \mathcal{F} a Zassenhaus family (of left ideals) of the ring R .) While the converse doesn't hold in general, we will examine examples of rings R for which the converse does hold, i.e. R has a Zassenhaus family \mathcal{F} of left ideals that can be used to construct a left R module M such that $R \subset M \subset \mathbb{Q}R$ and $End_{\mathbb{Z}}(M) = R$. (Received September 12, 2006)