

1023-20-682

**Luise-Charlotte Kappe\*** ([menger@math.binghamton.edu](mailto:menger@math.binghamton.edu)), State University of New York at Binghamton, Department of Mathematical Sciences, Binghamton, NY 13902-6000, and **Gabriela Mendoza** and **Michael Ward**. *Variations on a Theme by Desmond MacHale*.

In a 1981 paper "Minimal Counterexamples in Group Theory", Desmond MacHale states 47 conjectures, all known to be false, and asks for minimal counterexamples. In the spirit of MacHale we make the following conjecture:

Conjecture: For a given prime  $p$ , the elements of order dividing  $p$  in a group  $G$  always form a subgroup.

Obviously the conjecture is false. But what is the order of a minimal counterexample? Denoting with  $f(p)$  the order of a minimal counterexample, we have the following results.

Theorem 1: Let  $p$  be a prime. Then  $f(p) = p(p + 1)$  if and only if  $p = 2$  or a Mersenne prime.

Theorem 2. Let  $p$  be an odd prime which is not a Mersenne prime, then  $f(p) = \min(p(kp + 1), |PSL(2, p)|)$ , where  $k$  is the smallest integer such that  $kp + 1$  is a prime power.

Both cases can occur and because of the erratic behavior of  $k$ , not much can be said otherwise. (Received September 20, 2006)