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Thomas J Osler* (osler@rowan.edu), Mathematics Department, Rowan University, Glassboro, NJ 08028. *Euler's little summation formula and special values of the zeta function.*

In this note we present an elementary method of determining values of the Riemann zeta function $\zeta(z)$ for $z = 0, -1, -2, -3$, etc. We use a little known summation formula due to Euler. These values of the zeta function are commonly found in two ways. The first method uses the known values of the zeta function at positive even integers and the functional equation for the zeta function. A second method of finding these values of the zeta function is to use the theory of residues applied to a contour integral representation of $\zeta(z)$. We show that these values of the zeta function at 0 and the negative integers can be found without using either the functional equation or the special values of the zeta function at positive even integers. Nor do we need contour integrals and the theory of residues. We use only a rarely seen elementary summation formula due to Euler that we call "Euler's little summation formula". This little summation formula was found by Euler as an intermediate item in the derivation of his "big" result that we call today the Euler-Maclaurin summation formula. We show a simple derivation of the little summation formula using Taylor's theorem. (Received September 20, 2006)