Let $A$ be a separable $C^*$—algebra and $A^{**}$ its enveloping $W^*$—algebra. A result of Akemann, Anderson, and Pedersen states that if $(p_n)$ is a sequence of mutually orthogonal minimal projections in $A^{**}$ such that $\sum_k p_n$ is closed, $\forall k$, then there is a MASA $B$ in $A$ such that each $\phi_n|B$ is pure and has a unique state extension to $A$, where $\phi_n$ is the pure state of $A$ supported by $p_n$. We generalize this in two ways: It can be required that $B$ contain an approximate identity of $A$, and the countable discrete space underlying the above can be replaced by a totally disconnected space. We consider two types of type $I$ closed faces, both related to the above, atomic closed faces and closed faces with nearly closed extreme boundary (NCEB). A complement to Glimm’s theorem, which may or may not be new, arises from this. One specific question is whether an atomic closed face always has an “isolated point”. We give a counterexample for this and also show the answer is yes in the NCEB case. One of our examples is a type $I$ closed face which is isomorphic to a closed face of every non-type $I$ separable $C^*$—algebra and which is not isomorphic to a closed face of any type $I$ $C^*$—algebra. (Received September 21, 2006)