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George A Khachatryan* (georgek@uchicago.edu), 5550 S. Dorchester, Apt. 1203, Chicago, IL 60637. *The Theory of Lumpness: A Geometric Approach to the Expected Distance Between Two Points in a Probability Distribution Function in R^n* . Preliminary report.

An area of probability theory that appears to have been little explored is the expected distance between two points in a continuous probability density function in R^n , and in particular how local redistributions of probability density affect this expectancy. In this work, we develop a method which can be brought to bear on the problem.

We begin by defining a function, called the *lumpness*, on the collection of measurable sets in R^n which is homogeneous of degree 0 and, for sets of measure 1, corresponds to the expected distance between two points in the set. This allows for the geometric properties of a set (as opposed to the measure) that determine this expectancy to be investigated, since non-measure-preserving transformations can now be considered and standard methods applied. We differentiate this function and prove several results, including the smoothness of the level surfaces of the derivative and the convexity of their interiors.

We show that arbitrary continuous probability density functions (with convergent expected distance between two points) inherit these properties from sets, since they can be appropriately approximated by measurable sets.

Finally, several interesting open questions in the theory of lumpness are discussed. (Received September 26, 2006)