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Vladimir Koltchinskii. *Empirical graph Laplacian approximation of Laplace-Beltrami operators.*

Let M be a compact Riemannian submanifold of \mathbf{R}^m of dimension d and let X_1, \dots, X_n be a sample of i.i.d. points in M with uniform distribution. The random operators

$$\Delta_{h_n, n} f(p) := \frac{1}{nh_n^{d+2}} \sum_{i=1}^n K\left(\frac{p - X_i}{h_n}\right) (f(X_i) - f(p)), \quad p \in M$$

are studied, where $K(u)$ is the Gaussian kernel and $h_n \rightarrow 0$. Such operators can be viewed as graph laplacians (for a weighted graph with vertices at data points) and they have been used in the machine learning literature to approximate the Laplace-Beltrami operator of M , $\Delta_M f$ (divided by the Riemannian volume of the manifold). Several results are proved on a.s. and distributional convergence of the deviations $\Delta_{h_n, n} f(p) - \frac{1}{|\mu|} \Delta_M f(p)$ for smooth functions f both pointwise and uniformly in f and p . (Received September 13, 2006)