The general problem of interest here is: what kind of color patterns are forced to occur in every edge-coloring of some host graph satisfying certain constraints? This problem is motivated by the canonical Ramsey theorem and the Turán problem. As in the Turán problem, general techniques often fail for cycles, leaving the case for cycles particularly intriguing.

We survey some recent results and problems on cycles. In particular, we present a proof of the following conjecture of Axenovich, Jiang, and Tuza: for every $k$ there is a constant $\lambda_k$ depending only on $k$ such that for large $n$ if the edges of $K_n$ are colored such that at least $\lambda_k$ different colors appear at each vertex then we can always find a properly colored cycle of length exactly $k$ in this coloring. The value of $\lambda_k$ given in the proof is large. It remains as an interesting problem to find better bounds on $\lambda_k$. (Received September 19, 2007)