Define by $N_k$ the quotient poset of the Boolean lattice, $B_n$, under the relation “equivalence under rotation.” Griggs, Killian, and Savage proved that $N_p$ is a symmetric chain order for prime $p$. In this paper, we settle the question of whether this poset is a symmetric chain order for all $k$ by providing an algorithm that produces a symmetric chain decomposition. We accomplish this by modifying the idea of parenthesis matching from Greene and Kleitman. This allows us to take appropriate “middles” of the chains of a subset of the Greene- Kleitman SCD for $B_n$. We also prove additional properties of the resulting SCD and show that this settles some related conjectures. (Received July 23, 2007)