Carmichael’s conjecture that there is not a unique preimage for any value of Euler’s ϕ-function is well-known and widely believed. Since the unit group modulo $n$ has order $\varphi(n)$, one might be tempted to conjecture that there is never a unique preimage for the function $U$, which maps an integer $n$ to the isomorphism class of the unit group modulo $n$. However, this is false (consider $n = 24$). We give a lower bound on the number of counterexamples up to $x$, and we also show most numbers $n$ are not counterexamples. (Received September 20, 2007)