The $n$th cyclotomic polynomial, $\Phi_n$ is the unique monic polynomial whose roots are the primitive $n$th roots of unity. It is not difficult to show that these polynomials have integer coefficients and are irreducible. In this talk we will discuss some problems related to the size of coefficients of cyclotomic polynomials.

Let $A(n)$ be the maximum absolute value of a coefficient of $\Phi_n$. We say that $\Phi_n$ is flat if $A(n) = 1$. A cyclotomic polynomial has order $k$ if $n$ is the product of $k$ distinct odd primes. It is easy to show that any cyclotomic polynomial or order one or two is flat. We will present several interesting open problems related to cyclotomic polynomials of order three, four, and five. We will present some counterexamples to an upper bound for $A(pqr)$ conjectured by Beiter in 1971 which were recently discovered by Liu. For each pair of primes $p < q$ we will give infinitely many primes $r$ such that $\Phi_{pqr}$ is flat. We will discuss some results about the periodicity of $A(n)$ and explain how to use these to give several infinite families of flat cyclotomic polynomials of order four. We will also discuss attempts to determine whether there exist any flat cyclotomic polynomials of order five or higher. (Received October 17, 2007)