In work with Jan Bruinier, we extend work of Kohnen and Zagier and Waldspurger to obtain automorphic forms whose coefficients encode the vanishing and non-vanishing of quadratic twists of wgt 2 modular $L$-functions and their derivatives at the $s = 1$. Using Maass-Heegner divisors, we generalize Borcherds lifts to construct differentials of the third kind on modular curves. This construction, combined with work of Gross and Zagier, gives our results. As a special case, let $G$ be a weight 2 newform with prime level $p$ with the property that the sign of the functional equation of $L(G, s)$ is $-1$. We identify a Maass form

$$f(\tau) = \sum_{n \gg -\infty} c^+(n)q^n + \sum_{n < 0} c^-(n)H(2\pi n\nu)q^n$$

which enjoys the following:

1. If $\Delta < 0$ is fund. disc. for which $\left( \frac{\Delta}{p} \right) = 1$, then

$$L(G, \chi_{\Delta}, 1) = 32\|G\|^2\|g\|^2\pi^2 \sqrt{\Delta} : c_g^-(\Delta)^2.$$  

2. If $\Delta > 0$ is a fund. disc. for which $\left( \frac{\Delta}{p} \right) = 1$, then $L'(G, \chi_{\Delta}, 1) = 0$ if and only if $c_g^+(\Delta)$ is algebraic.

We obtain the general result for arbitrary levels and signs. (Received September 10, 2007)