The nonlinear integral equation $P(x) = \int_{\alpha}^{\beta} dy \, w(y) P(y) P(x + y)$ is investigated. It is shown that for a given function $w(x)$ the equation admits an infinite set of polynomial solutions $P(x)$. For polynomial solutions, this nonlinear integral equation reduces to a finite set of coupled linear algebraic equations for the coefficients of the polynomials. Interestingly, the set of polynomial solutions is orthogonal with respect to the measure $x w(x)$. The nonlinear integral equation can be used to specify all orthogonal polynomials in a simple and compact way. This integral equation provides a natural vehicle for extending the theory of orthogonal polynomials into the complex domain. Generalizations of the integral equation are discussed.

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