Type A Cherednk algebras $H_c$, which are particular deformations of the twisted group ring of the $n$-th Weyl algebra by the symmetric group $S_n$, form an intriguing class of algebras with many interactions with other areas of mathematics. In earlier work the authors proved a sort of Beilinson-Bernstein equivalence of categories, thereby showing that $H_c$ (or more formally its spherical subalgebra $U_c$) forms a noncommutative deformation of the Hilbert scheme $Hilb(n)$ of $n$ points in the plane. One question that arose in that work was whether the characteristic varieties of irreducible $U_c$-modules are equidimensional subshemes of $Hilb(n)$.

In this lecture we will prove this conjecture and give various applications. (Received September 18, 2007)