Supercharacter theories of cyclic $p$-groups.

If $\mathcal{K}$ is a partition of a finite group $G$, there sometimes exists a compatible partition $\mathcal{X}$ of the irreducible characters of $G$, along with a character $\chi_X$ for every $X \in \mathcal{X}$ with the elements of $X$ as its irreducible constituents, so that each $\chi_X$ is constant on each $K \in \mathcal{K}$ and $|\mathcal{X}| = |\mathcal{K}|$. If $\{1\} \in \mathcal{K}$, then P. Diaconis and M. Isaacs have called such an ordered pair $(\mathcal{X}, \mathcal{K})$ a supercharacter theory.

We describe the set of all supercharacter theories (up to scaling) of the cyclic group of order $p^n$ for an odd prime $p$, and we show that its cardinality is a polynomial in $d$ of degree $n$, where $d$ is the number of divisors of $p - 1$. (Received September 20, 2007)