Let $X_n$ be a finite set with $n$ elements. The semigroup $P_n$ of partial transformations on $X_n$ consists of all functions whose domain and image are included in $X_n$, with composition as the operation. It includes as its subsemigroups the symmetric group $S_n$, the full transformation semigroup $T_n$, and the symmetric inverse semigroup $I_n$, where $S_n$, $T_n$, and $I_n$ consist, respectively, of the permutations, full transformations, and partial 1-1 transformations on $X_n$. The symmetric group $S_n$ is the group of units of $P_n$, $T_n$, and $I_n$.

A subsemigroup $S$ of $P_n$ is called $S_n$-normal if it is closed under conjugations by permutations, that is, if for all $a \in S$ and $g \in S_n$, $g^{-1}ag \in S$. This concept generalizes the well-known notion of a normal subgroup. The $S_n$-normal subsemigroups of $T_n$ were determined in 1976; and the $S_n$-normal subsemigroups of $I_n$ were described in 1995. We complete the picture by providing a complete classification of the $S_n$-normal subsemigroups of $P_n$. In contrast with the classifications for $T_n$ and $I_n$, the problem of classifying the $S_n$-normal subsemigroups of $P_n$ does not reduce to finding the $S_n$-normalizers ($S_n$-normal semigroups generated by one element). (Received September 18, 2007)