We consider the following quasilinear elliptic problem,

\[(P) \quad -\Delta_p u = \lambda |u|^{p-2}u + f(x) \quad \text{in } \Omega; \quad u = 0 \quad \text{on } \partial \Omega,\]

and a strictly cooperative system of such equations. Here, \(\Omega\) is a smooth, open bounded domain in \(\mathbb{R}^N\) and \(\Delta_p u\) denotes the \(p\)-Laplace operator, \(1 < p < \infty\). The real number \(\lambda\) is a spectral parameter. Given a function \(f \in L^\infty(\Omega)\), \(f \geq 0\), we investigate the existence, uniqueness, and positivity of a weak solution \(u \in W^{1,p}_0(\Omega)\) to problem \((P)\). In the first part we concentrate on the existence and simplicity of the first (smallest) eigenvalue \(\lambda_1\) for both, the single equation \((P)\) and a strictly cooperative system of such equations. We apply a method using the monotonicity and the part metric. In the second part we discuss the strong comparison principle for problem \((P)\). For \(1 < p \leq 2\) we use monotone dynamics of a cooperative system of two ODE’s to show this principle for the radially symmetric problem in a ball, \(f \geq 0\), and \(\lambda < \lambda_1\). For \(2 < p < \infty\) we give a simple counterexample for such a problem if \(\lambda\) is large negative. (Received September 18, 2007)