Multicusped solitons of the Camassa-Holm equation.

The eponymous Camassa-Holm (CH) equation,

$$u_t + 2\kappa^2 u_x + 3uu_x - u_{xxx} = 2u_xu_{xx} + uu_{xxx}, \quad \kappa > 0,$$

has proved to be of considerable and enduring interest since it resurfaced as a model for shallow-water waves. Over the last decade or so, the equation has attracted a substantial literature, much of which has been devoted to establishing its integrable credentials. Apart from the familiar classical solitons, the CH equation admits nonanalytic ‘peaked’ solitons: the so-called peakons and cuspons. The peakon solitary wave possesses a peak that is a ‘corner’ (a finite discontinuity in the slope), whilst the cuspon has infinite and opposite slope at its apex (a cusp).

Yet despite their being piecewise analytic, cuspons interact in typical soliton fashion both amongst themselves and, remarkably, with their smooth soliton counterparts. In this talk, explicit multiple cuspon-soliton solutions of the CH equation are reported in parametric form. The dynamics of the two-wave interactions, in particular, are discussed and some unanswered questions concerning these waveforms are addressed. Examples of two and three cuspon-soliton solutions are presented illustrating the different parameter regimes. (Received September 21, 2007)