The piston dispersive shock wave problem.

The one-dimensional piston shock problem is a classical result of shock wave theory and is discussed in many textbooks. The analogous problem for a dispersive fluid (e.g. a Bose-Einstein condensate or light propagation through a nonlinear, defocusing medium) is considered theoretically. In contrast to the well known theory of classical shock waves in a compressible fluid where dissipation plays the dominant role in regularizing the shock solution and a strong limit exists, the propagation of a dispersive shock wave (DSW) through a dispersive fluid requires a dispersive regularization where only a weak limit exists. The dispersive regularization of a nonlinear Schrödinger type equation with a moving step potential (the piston) is considered. Asymptotic solutions are calculated using 1-phase Whitham averaging theory for a piston moving with uniform speed into a dispersive fluid at rest. These asymptotic results agree quantitatively with numerical simulations. It is shown that the behavior of the asymptotic solutions is quite different from their classical counterparts. In particular, the shock structure depends on the speed of the piston. (Received September 14, 2007)