Let $F$ be a smooth foliation of a compact manifold $M$. A hyperbolic attractor $F$ is a minimal set $K$ such that there is positive Lyapunov exponent for the linear holonomy of $F$ restricted to $K$. Of particular interest is the case where $K$ is an exceptional minimal set which is transversally modeled on a connected continua - these are called exotic minimal sets. We discuss the relation between foliation entropy, the Lyapunov exponents of $F$ on $K$, and the geometry of $K$. A general construction of examples of foliations with exotic minimal sets is described for codimensions greater than one. (Received September 20, 2007)