For every element $f$ in the integral group ring of the discrete Heisenberg group $\Gamma$ there is associated, via Pontryagin duality, an action of $\Gamma$ by automorphisms of a compact abelian group. In joint work with Klaus Schmidt, we determine for which $f$ this action is ergodic, or mixing, or expansive, and discuss the entropy of this action. There are exact calculations of entropy in some simple cases, but in general computing entropy leads to some fascinating open problems about products of noncommuting matrices. These ideas can also be used to define a natural notion of Mahler measure for polynomials in noncommuting variables, and to generate a host of open problems as well. (Received August 01, 2007)