Example of a mean ergodic $L^1$ operator with the linear rate of growth.

The rate of growth of an operator $T$ satisfying the mean ergodic theorem (MET) cannot be faster than linear. It was recently shown [7] that for every $\gamma > 0$, there are positive $L^1[0, 1]$ operators $T$ satisfying MET, with $\lim_{n \to \infty} \frac{\|T^n\|}{n^{1-\gamma}} = \infty$. In the class of positive $L^1$ operators this is the most one can hope for in the sense that for every such operator $T$, there exists a $\gamma_0 > 0$, such that $\limsup_{n} \frac{\|T^n\|}{n^{1-\gamma_0}} = 0$. In this note we construct an example of a nonpositive $L^1$ operator with the highest possible rate of growth, that is $\limsup_{n \to \infty} \frac{\|T^n\|}{n} > 0$. (Received September 20, 2007)