We develop a new class of scattering matrices on graphs which have equal transmission probabilities and no reflection amplitude. This follows previous generalizations of spectral problems on quantum graphs where scattering matrices defined by an operator on the graph are replaced by particular classes of unitary matrices. Choosing matrices which do not allow backscattering implies the set of periodic orbits summed over in the trace formula is the same as that appearing in the definition Ihara zeta function on the corresponding combinatorial graph. We derive properties of equi-transmitting graphs where the associated doubly stochastic Markov matrices have a large spectral gap and hence the spectral statistics are conjectured to converge rapidly to those of random matrices. Equi-transmitting matrices can be generated from skew-Hadamard matrices and symmetric equi-transmission matrices can be constructed in dimension \( p + 1 \) where \( p \equiv 1 \mod 4 \) is a prime using Dirichlet characters. (Received July 11, 2007)