Given any $\varepsilon > 0$, it is possible to construct a compact set in the plane of measure $< \varepsilon$, containing a line segment of unit length in every direction. Putting $\varepsilon = 1/k$ and taking the intersection over all $k$ yields a Besicovitch-Kakeya set. Examples are well known, as is the fact that this cannot be done for all rotations of $k$-planes in $\mathbb{R}^n$ for $1 < k \leq n - 1$.

Let $E(x)$ denote the characteristic function of a compact set in $\mathbb{R}^n$ and $\hat{E}(\theta,t)$ its Radon transform. For a Besicovitch-Kakeya set in $\mathbb{R}^2$, the variation of $\hat{E}$ is $\geq 2$ for every direction $\theta$. We obtain estimates of a sets measure in terms of the variation of the derivative of its Radon transform of order $< n/2 - 1$. Upper and lower bound estimates are obtained for all $n \geq 3$; evidently, the inequality only goes one way for $n = 2$. The estimates are best possible for $n = 2$ and 3. The analysis is also carried out for absolutely integrable functions with compact support. (Received May 14, 2007)